Ideas of optimization: from saving a drowning child, to managing an investment portfolio, to designing the F-22 Raptor

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Quotes

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- "Mathematics is a game played according to certain simple rules with meaningless marks on paper." – David Hilbert
- "Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something entirely different." – Johann Wolfgang von Goethe (Maxims and Reflexions, 1829)

Light optimizes: Reflection

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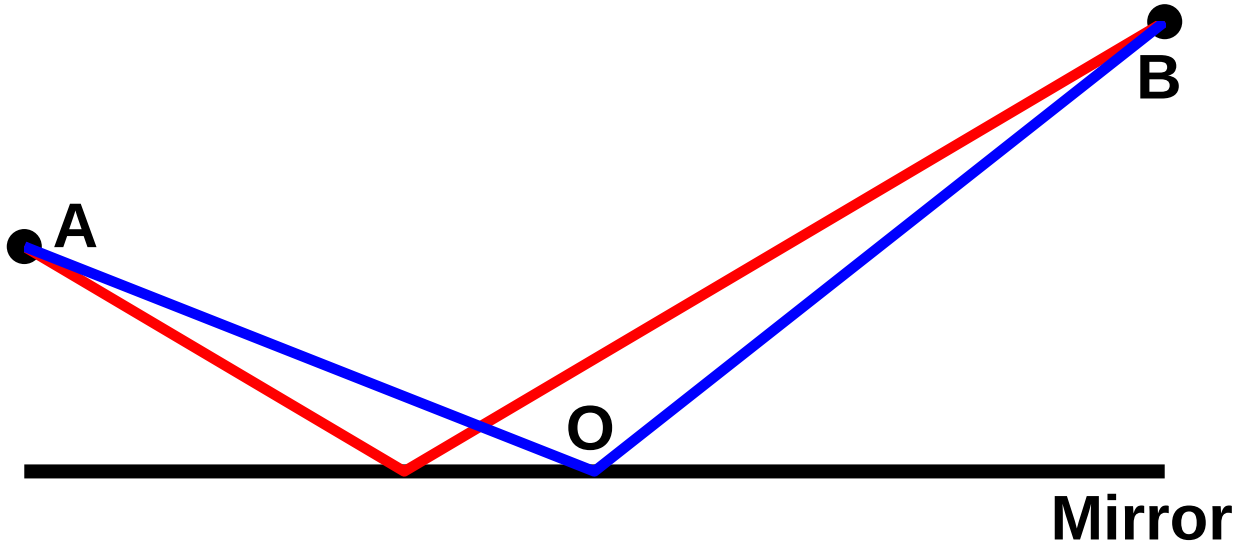
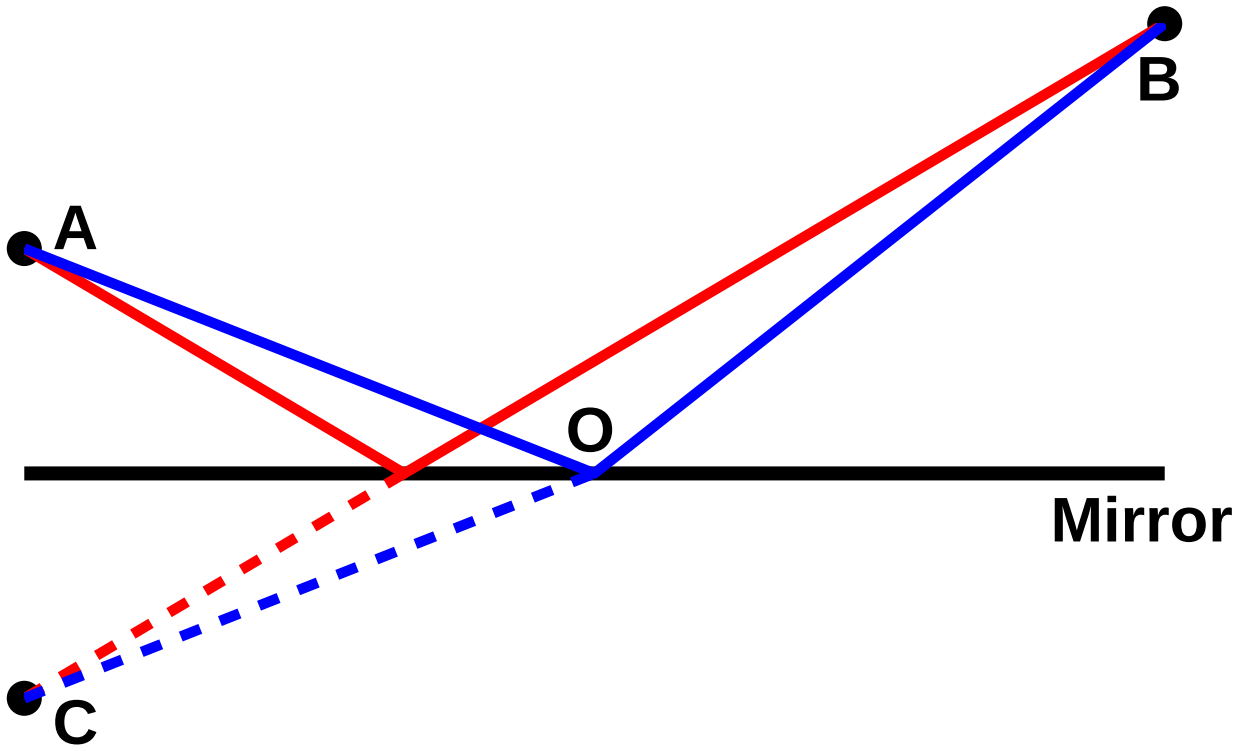


Which path? Shortest one!

Light optimizes: Reflection: a proof by Hero



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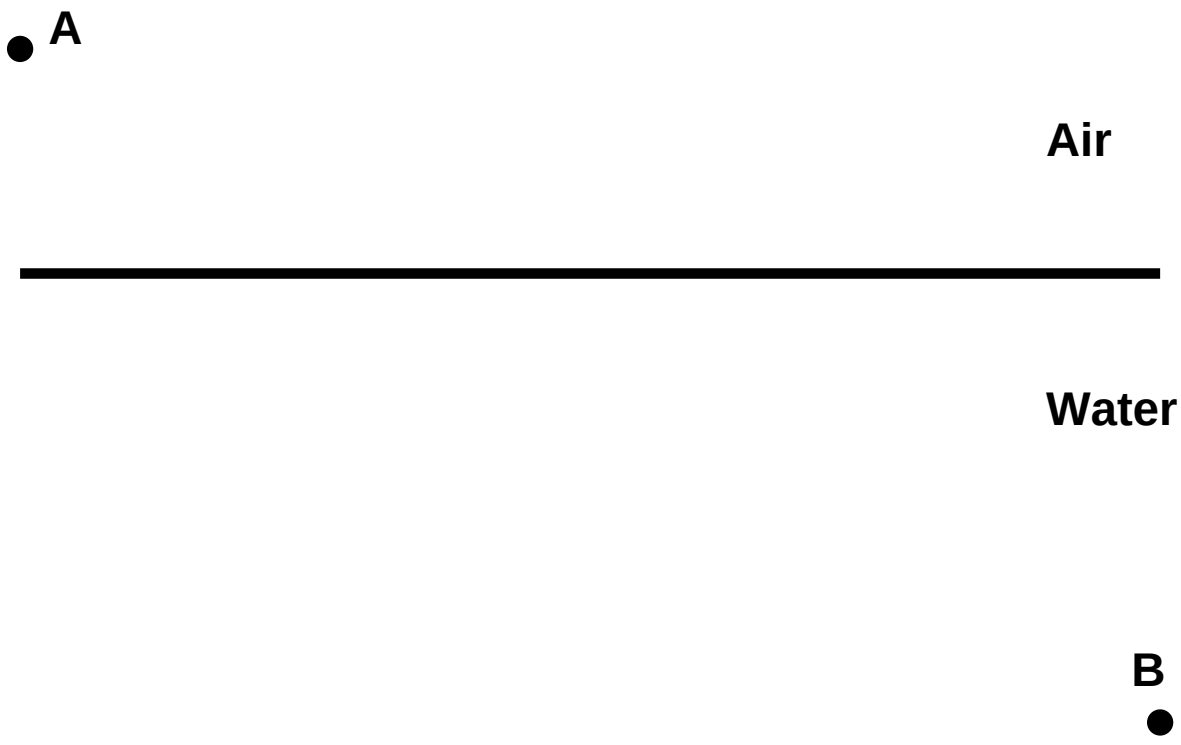
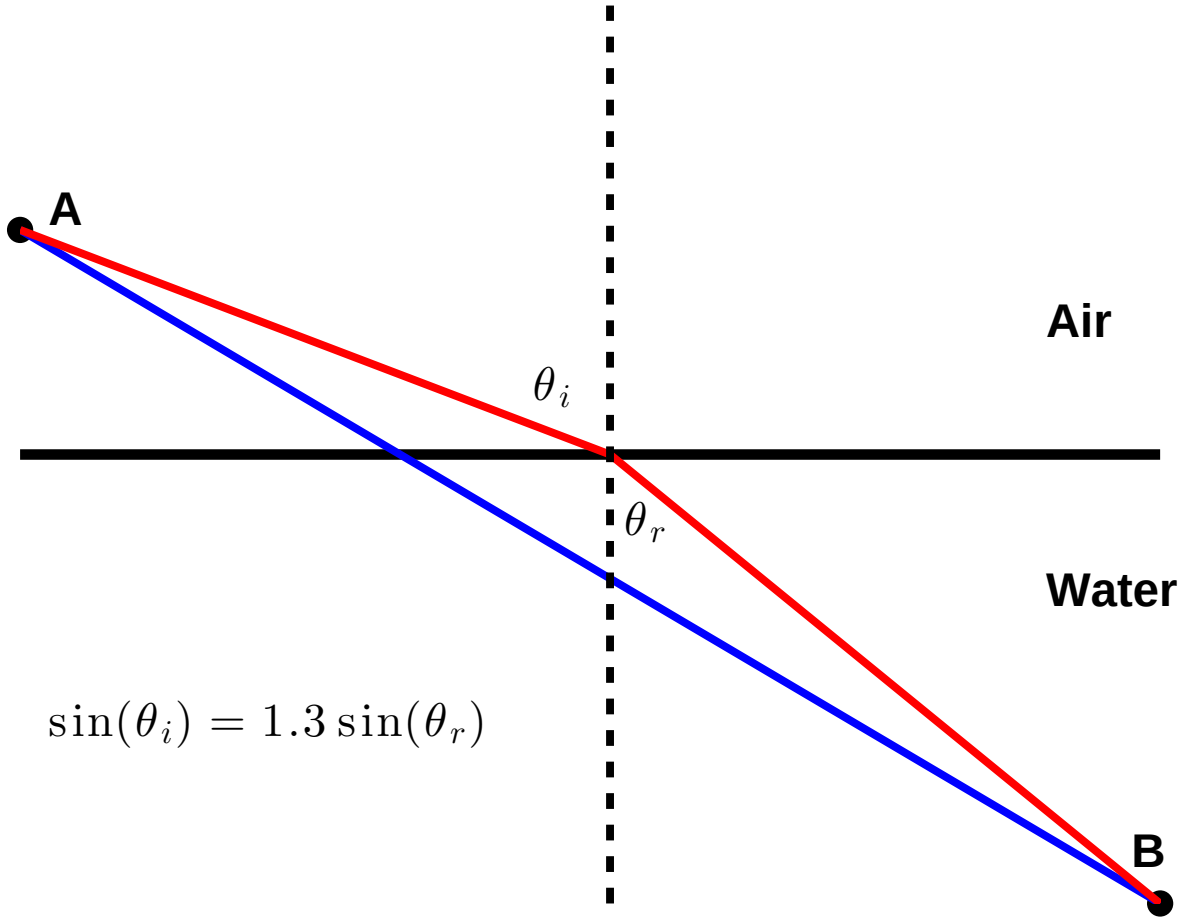
Light optimizes: Rarefaction



Light optimizes: Rarefaction



Fermat: Light does NOT take the shortest path, but the fastest path

We people optimize

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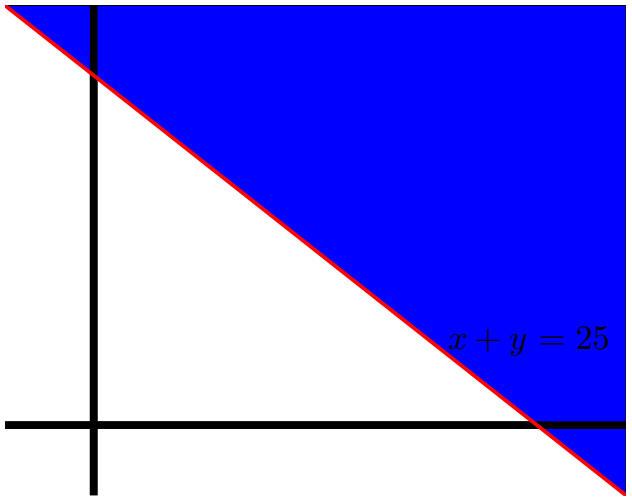
Linear Programing: Investment Portfolio Management

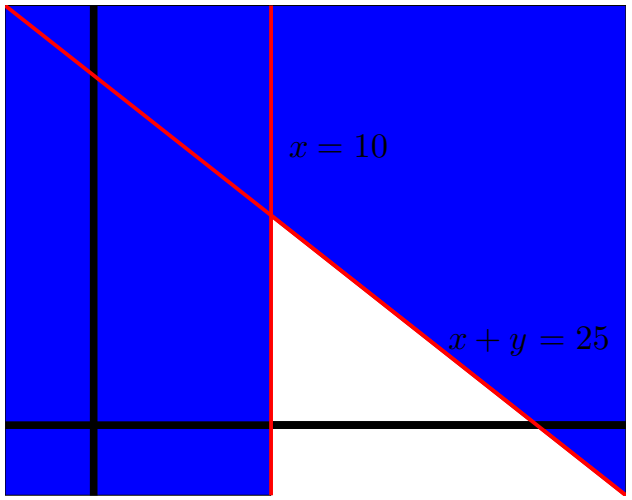
- A bank has to set aside $25 million for loans to home builders.
- Allocate at least $10 million for luxury condos.
- Required by government: at least one third of its total loans should be allocated to low-income housing.
- Returns on condos is 12% and returns on low-income housing is 10%.

$$\text{Maximize} \quad p = 0.12x + 0.10y$$

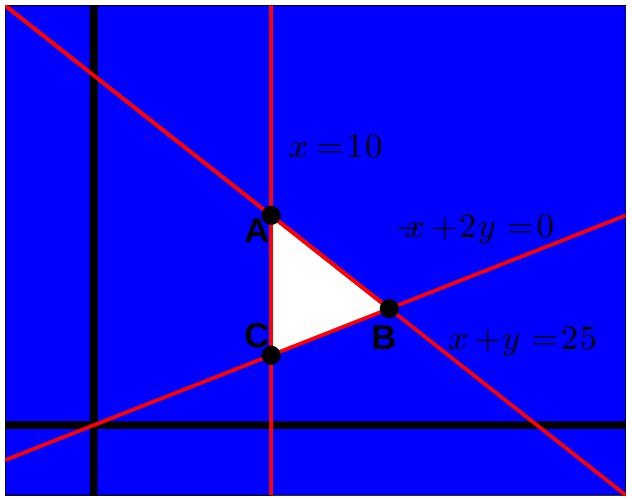
$$\text{Subject to} \quad x + y \leq 25$$



$$\begin{aligned} \text{Maximize} \quad & p = 0.12x + 0.10y \\ \text{Subject to} \quad & x + y \leq 25 \\ & x \geq 10 \end{aligned}$$



$$\begin{aligned} \text{Maximize} \quad & p = 0.12x + 0.10y \\ \text{Subject to} \quad & x + y \leq 25 \\ & x \geq 10 \\ & -x + 2y \geq 0 \end{aligned}$$



Points	Lines through points	Coordinates	$p = 0.12x + 0.10y$
A	$x + y = 25$ $x = 10$	$(10, 15)$	2.7
B	$x + y = 25$ $-x + 2y = 0$	$(50/3, 25/3)$	2.833
C	$x = 10$ $-x + 2y = 0$	$(10, 5)$	1.7

How does F22 hide from radar: optimal shape

(Re Ez)

(Re Ez)